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**AIRPORT AND AIRLINE COMPETITION IN A MULTIPLE  
AIRPORT REGION: AN ANALYSIS BASED ON  
THE NESTED LOGIT MODEL<sup>1</sup>.**

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**ABSTRACT**

In a multiple airport region airlines compete with both other airlines operating from the same airport and airlines operating from alternative airports. In this paper symmetric equilibrium airfares and frequencies are derived for airlines operating from the same airports and for airlines operating from different airports. These equilibria are shown to be unique. Next, airport authorities are introduced as independent agents and equilibrium airport taxes, airfares and frequencies are derived and shown to be unique. Some simplifying assumptions are necessary to be able to derive these equilibria. We comment on the possibilities of relaxing these assumptions.

**INTRODUCTION**

In this paper we use a logit demand system to derive equilibrium airfares, frequencies and airport taxes in a multiple airport region. In a previous paper (Pels et al., 1998,1), it was argued, based on a deterministic model, that an equilibrium between airports and airlines may not exist; the airport's reaction curve may not exist over a specific airfare interval. Moreover, only a price equilibrium was considered. In

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this paper we include frequencies and investigate whether local airline equilibria (between airlines operating from the same airports), global airline equilibria (between all airlines in the region) and global equilibria (between all airlines and airports) are unique. To be able to derive the desired equilibria some simplifying assumptions are needed. We also comment on the effect of a relaxation of these assumptions. The main purpose of the paper is to derive competitive symmetric equilibrium airfares, frequencies and airport taxes and to show the uniqueness of the equilibrium. The paper has the following structure. In Section 2 a concise review of the literature on discrete choice models and their use in the analysis of passenger preferences concerning airports and airlines will be given. In Section 3 a competition model is developed and the equilibria are derived. Section 4 offers some conclusions.

## LITERATURE REVIEW

### *Disaggregate Choice Models*

Passengers traveling by air need to make a number of decisions; which airport to use (when there are alternative airports available), which airline to use etcetera. In the literature, such decisions are usually modeled using the multinomial logit model (MNL); see e.g. Bondzio (1996) for an analysis of airport choice in Germany, Brooke et al. (1994) and Caves et al. (1991) for an analysis of airport choice in the UK and Harvey (1987) for an analysis of airport choice in the San Francisco Bay Area. In all studies, access time to the airport was found to be an important determinant of airport choice. Brooke et al. (1994) and Caves et al. (1991) also find the airfare to be of influence, while both Bondzio (1996) and Harvey (1987) omit the airfare; Harvey argues that there appeared to be more variation among fare classes on a given flight to a particular destination than among different flights to that destination or airport. Moreover, information on the airfare actually paid by passengers is scarce. Frequency of service was found to be a significant determinant of airport choice by all above mentioned authors. Hansen (1990) employed a MNL to estimate market shares of airlines in origin-destination markets. The estimated market share was then used as input for an airline competition model. For direct services, the explanatory variables were the airfare and the (log of) frequency of service. Ndoh et al. (1990) found that a nested multinomial logit model (NMNL) is statistically to be preferred to a MNL. Moreover, based on a likelihood ratio test Ndoh et al. (1990) conclude the MNL structure violates the IIA assumption. Pels et al. (1998,2) also use a NMNL for an analysis of airport choice in the San Francisco Bay Area.

### *Multi-Product Firms*

Anderson et al. (1996) use a nested logit demand model to describe multiproduct firms. In their study, consumers have to select both firms and products, and firms have to decide i) whether to enter the market, ii) how many products

(alternatives) to produce and iii) the level of prices. Anderson et al. show a unique symmetric price equilibrium exists. When there are several airlines competing on a particular route, passengers have to decide which airline to use (given the choice of departure airport). In other words, from the departure airport there are several products (flights) available to a particular destination. These flights however, are delivered by independent airlines. So rather than having a single firm making all three decisions mentioned above, these decisions are made by a number of independent agents. Concluding, a NMNL, which, in the case of airport choice, Ndoh et al. (1990) found to be superior to the MNL, can be used to describe airport and airline competition in a multiple airport region. The choice of nest (airport), will depend on airport characteristics (access time, tax) and the expected utility of using the alternatives (flights or airlines) available from each nest. The choice of flight (route, airline) can be modeled using a specification like in e.g. Hansen (1990). This model will be developed in the next Section.

### THEORETICAL CONSIDERATIONS

In this Section the discrete choice model for airport choice in a multiple airport region is formulated and some theoretical considerations on competition between airports are presented. Subsection 3.1 deals with the discrete choice model, Subsection 3.2 deals with the airport's maximization problem (and competition between airlines operating from the same airport). Finally, Subsection 3.3 deals with the airport's maximization problem and competition between both airports and airlines.

#### *The Passenger Discrete Choice Model*

Suppose a traveler  $i$  has decided to travel by air to a particular destination. The traveler then has to make two choices; one for the origin airport (nest) and one for the airline. These choices are based on the (maximum) (in)direct utility the passenger derives from using a particular (combination of) departure airport  $d$  and airline  $l$ . The choices can be made sequentially or simultaneously. In what follows we assume the choices are made sequentially; we will comment on the implications of a simultaneous choice.

Assume a passenger first chooses an airport and then an airline. Airline  $l$  offers  $f_l$  flights to a particular destination. Each particular flight  $j$  results in a utility  $V_{lj}$ . The average utility over all flights to a particular destination is  $\bar{V}_l = \frac{1}{f_l} \sum_j V_{lj}$ . Then, if the utilities of all elemental flights  $j$  are IID (which implies  $\bar{V}_l = V_{lj}$  for all  $j$ ), it can be shown that the distribution of the aggregate alternative  $l$  approaches the Gumbel distribution and as a result the total utility derived from airline  $l$  can be

written as  $U_l = \bar{V}_l + \beta_f \ln(f_l) + \varepsilon_l$  (see Ben-Akiva and Lerman, 1987). The utility of using an airline  $l$  is therefore determined by the average airfare ( $p_l$ ) and the log of the frequency of service ( $f_l$ ). The utility of using airport  $d$  depends on the airport tax  $tax_d$ , the access time to the airport  $t_d$  and the maximum expected utility  $\bar{V}_d$  of the alternatives in the choice set (of airlines) available from each departure airport  $d$ . Then the probability that a combination (departure airport  $d$ , airline  $l$ ) is chosen can be expressed as<sup>3</sup>:

$$P(l, d) = P(l|d)P(d) \quad (1)$$

$$P(l|d) = \frac{\exp\left(\frac{\alpha_l - \alpha_p p_{l,d} + \alpha_f \ln(f_{l,d})}{\mu_2}\right)}{\sum_{l'} \exp\left(\frac{\alpha_{l'} - \alpha_p p_{l',d} + \alpha_f \ln(f_{l',d})}{\mu_2}\right)} \quad (2)$$

$$P(d) = \frac{\exp\left(\frac{\beta_d - \beta_t tax_d + \beta_a \ln(t_d) + \bar{V}_d}{\mu_1}\right)}{\sum_{d'} \exp\left(\frac{\beta_{d'} - \beta_t tax_{d'} + \beta_a \ln(t_{d'}) + \bar{V}_{d'}}{\mu_1}\right)} \quad (3)$$

with

$$\bar{V}_d = \mu_2 \ln \left( \sum_l \left( \frac{\alpha_l - \alpha_p p_{l,d} + \alpha_f \ln(f_{l,d})}{\mu_2} \right) \right) \quad (4)$$

The parameters  $\alpha_p$ ,  $\alpha_f$ ,  $\beta_t$  and  $\beta_a$  are assumed to be greater than zero. The parameter  $\mu_2$  represents the degree of heterogeneity of airlines (flights) within (from) an airport. The closer  $\mu_2$  is to 0, the higher the degree of substitutability between airlines.  $\mu_1$  is a measure of heterogeneity between airports. It is necessary that  $\mu_1 > \mu_2$  (see e.g. Ben-Akiva and Lerman, 1987 and Anderson et al., 1996). This means that airlines (flights) operating (originating) from the same departure airports are closer

<sup>3</sup> We assume the random utility component derived from using the combination  $(l, d)$  is (independent and identically) Gumbel distributed with scale parameter  $\mu_2$ . The random utility derived from using departure airport  $d$  is distributed so that  $\max_d U_{l,d}$  is Gumbel distributed with scale  $\mu_1$ , where  $U_{l,d}$  is the total utility of using the combination  $(l, d)$ .

substitutes than airlines (flights) operating (originating) from different departure airports.

In the next Subsections the nested logit model, as specified in equations 1,2,3 and 4, will be used to model airport and airline competition in a multiple airport region.

#### *The Airline's Maximization Problem*

An airline will operate a route to a particular destination from departure airport  $d$ , if it can generate non-negative profits. The demand for services from the combination  $(l,d)$  can be expressed as

$$Q_{l,d} = P(l,d)N \quad (5)$$

where  $N$  is the total number of passengers in the system and  $P(l,d)$  is defined in equation (1). The airline's profits obtained from operations (out) of airport  $d$  on a certain route are:

$$\pi_l = (p_l - c_l)Q_{l,d} - k_l f_l - K_l \quad (6)$$

where  $p_l$  is the price (airfare on that route),  $c_l$  is the (constant) marginal cost per passenger,  $f_l$  is frequency on that route,  $k_l$  is the constant marginal cost per flight and  $K_l$  is the fixed cost. The airline has to determine both the optimal fare and the frequency.

Maximizing profits with respect to the airfare yields

$$\left( \frac{\partial P(l,d)}{\partial p_l} = \alpha_p P(l,d) \left( \frac{P(l,d)(P(d)-1)}{\mu_1} + \frac{P(l,d)-1}{\mu_2} \right) \right):$$

$$\begin{aligned} \frac{\partial \pi_l}{\partial p_l} &= Q_{l,d} + (p_l - c_l) \frac{\partial Q_{l,d}}{\partial p_l} = 0 \Leftrightarrow \\ p_l - c_l &= \frac{\mu_1 \mu_2}{\alpha_p (\mu_2 P(l,d)(1 - P(d)) + \mu_1 (1 - P(l,d)))} \end{aligned} \quad (7)$$

where  $\frac{\partial \pi_l}{\partial p_l} = 0$  is the airline's best airfare response function.

Maximizing profits with respect to the frequency of service yields:

$$\frac{\partial \pi_l}{\partial f_l} = (p_l - c_l) \frac{\partial Q_{l,d}}{\partial f_l} = k \Leftrightarrow$$

$$f_l = \frac{\alpha_f Q_{l,d} (p_l - c_l) (\mu_2 P(l|d)(1 - P(d)) + \mu_1 (1 - P(l|d)))}{k \mu_1 \mu_2} \quad (8)$$

Equation 8 gives an expression for the optimum frequency at any given airfare; there is a whole range of airfares for which optimal frequencies can be determined. However, these airfares are not necessarily optimal. Substituting for the optimal airfare yields:

$$f_l = \frac{\alpha_f P(d)P(l|d)}{\alpha_p k} N \quad (9)$$

Equations 7 and 9 give the optimal airfare and frequency for an airline  $l$ . At any other combination  $(f_l, p_l)$  the optimum will not be reached (i.e. only for one particular value of  $p$  the solution to the airline's best response frequency function (equation 8) maximizes profits).

Let there be  $L$  airlines operating from airport  $d$ , with  $c_l = c_f$  and  $k_l = k_f$ . Then it is possible to find a symmetric equilibrium at which all airlines operating out of airport  $d$  have the same frequency of service and airfare, which is shown in Proposition 1.

**Proposition 1**

Given  $L$  airlines, with identical marginal cost per passenger  $c$  and identical marginal cost per flight  $k$ , operating out of airport  $d$ , and a frequency elasticity of (local) demand which is smaller than 1, there exists a unique symmetric equilibrium. The equilibrium price is given by:

$$p_d^* = c + \frac{L\mu_1\mu_2}{\alpha_p (\mu_1(L-1) + \mu_2(1-P(d)))} \quad (10)$$

and the equilibrium frequency is given by:

$$f_d^* = \frac{1}{L} \frac{\alpha_f}{\alpha_p} \frac{P(d)N}{k} \quad (11)$$

For a proof we refer to Appendix 1. Only the equilibrium airfare is directly dependent on the heterogeneity between both airlines operating from the same airport and airlines operating from different airports. As the heterogeneity increases so does the airfare. Furthermore, as the marginal probability  $P(d)$  increases, so does the equilibrium airfare. Finally, the equilibrium airfare is decreasing in  $L$ . Caves et al. (1991), using the daily frequency as an explanatory variable in a MNL found frequency elasticities between 0.048 and 0.738, depending on the passenger type (leisure, business, resident or foreign) and airport (Heathrow, Stansted or Gatwick).

The equilibrium expressions for  $p_d^*$  and  $f_d^*$  only show how they are determined at airport  $d$ , given  $P(d)$ . As  $P(d)$  changes (e.g. due to a change in  $p_{d'}$  or  $f_{d'}$ ,  $p_d^*$  and  $f_d^*$  change accordingly. What is important, is that all airlines operating from airport  $d$  will still have equal prices and frequencies. This allows for a simplification of  $\tilde{V}_d$  to  $\mu_2 \ln(L) - \alpha_p p_d^* + \alpha_f \ln(f_d^*)$ . For airport(s)  $d'$  similar expressions for  $p_{d'}^*$ ,  $f_{d'}^*$  and  $\tilde{V}_{d'}$  can be derived. Then, using Proposition 1, it is possible to find a system wide symmetric equilibrium as is shown in Proposition 2.

### Proposition 2

Let there be  $D$  airports, each of which accommodates  $L$  airlines with identical marginal cost per passenger  $c$  and identical marginal cost per flight  $k$ . Then there exists a unique symmetric price-frequency equilibrium. The equilibrium price is given by:

$$p^* = c + \frac{LD\mu_1\mu_2}{\alpha_p(\mu_1 D(L-1) + \mu_2(D-1))} \quad (12)$$

and the equilibrium frequency is given by:

$$f_d^* = \frac{1}{D} \frac{\alpha_f}{\alpha_p} \frac{N}{k} \quad (13)$$

Only the airfare is dependent on the heterogeneity between airlines operating from both the same airport and different airports. Both the equilibrium airfare and frequency are decreasing in  $D$  (and  $L$ ). This is an equilibrium between all the airlines in the system, taking the airport authorities' behavior as given. Note that although airlines charge the same prices and offer the same frequencies, airports do not necessarily have the same airport taxes and access times. As is shown in Appendix A, for the symmetric equilibrium to exist it is necessary that  $\Delta \equiv -\beta_1(\text{tax}_{d'} - \text{tax}_d) - \beta_a(\ln(t_{d'}) - \ln(t_d)) = 0$ . This relationship may be satisfied for  $\text{tax}_{d'} \neq \text{tax}_d$  and  $\ln(t_{d'}) \neq \ln(t_d)$ . Hence an airport with excellent accessibility may charge higher airport taxes than an airport with lesser accessibility. The relation  $\Delta=0$



says nothing about whether the airport taxes are optimal from the airports point of view. Therefore, in the next Subsection airports will be introduced as independent agents.

### *The Airport's Maximization Problem*

In this Subsection we will describe how airports in a multiple airport system can compete for passengers. Airport  $d$  has a market share  $P(d)$ , defined as in Subsection 3.1. Clearly, this probability also depends on the airline's optimal fares and frequencies.

Passenger demand for flights out of an airport  $d$  is given by  $P(d)$ . Let the airport's optimization problem be (see also Oum et al., 1996):

$$\begin{aligned} \max_{tax_d} \int P(d) dtax_d + (tax_d - mc_d) NP(d) - rK_d - g\left(\frac{NP(d)}{K_d}\right) \\ s.t. \Pi_d \equiv (tax_d - mc_d) NP(d) - rK_d \geq 0 \end{aligned} \quad (14)$$

the airport maximizes social welfare with respect to the airport tax  $tax_d$  under a cost recovery constraint. We assume the marginal costs per passenger  $mc_d$  are constant.  $rK_d$  is the capital cost of airport  $d$  (where  $K_d$  is the airport's capital stock) and  $g(\cdot)$  is a congestion cost function;  $\frac{\partial g(\cdot)}{\partial P(d)} > 0$ . The first order conditions are:

$$\begin{cases} (tax_d - mc_d) - \frac{\partial g\left(\frac{NP(d)}{K_d}\right)}{\partial tax_d} - \beta_{tax} P(d)(1 - P(d)) + \lambda \frac{\partial \Pi_d}{\partial tax_d} = 0 \\ \lambda \Pi_d = 0; \Pi_d \geq 0, \lambda \geq 0. \end{cases} \quad (15)$$

where  $\frac{\partial \Pi_d}{\partial tax_d} = P(d) - (tax_d - mc_d) \frac{\beta_{tax}}{\mu_1} P(d)(1 - P(d))$ . Then, if the cost recovery constraint is binding ( $\lambda > 0$ ):

$$tax_d = mc_d + \frac{rK_d}{NP(d)} \quad (16)$$

if  $\frac{rK_d}{NP(d)} > \frac{\partial \left( \frac{NP(d)}{K_d} \right)}{\partial tax_d}$ . Hence, if the capital costs exceed the marginal congestion costs, the airport sets its tax at average cost level. The tax curve is decreasing in  $P(d)$ .

Let  $L=2$ ,  $Kap_d = Kap_{d'}$ ,  $mc_d = mc_{d'}$  and  $t_d = t_{d'}$ . Then we can derive a symmetric equilibrium, which is shown in proposition 3.

**Proposition 3**

Given the conditions necessary for Propositions 1 and 2 and that the airport's profit function is non-decreasing in the airport tax, the following symmetric equilibrium is unique:

$$tax_d = mc_d + \frac{rK_d D}{N} \quad (17)$$

$$p^* = c + \frac{D\mu_i}{\alpha_p(D-1)} \quad (18)$$

$$f^* = \frac{I \alpha_f N}{D \alpha_p k} \quad (19)$$

See Appendix A for a proof. The equilibrium airport tax is increasing in the number of airports; as the number of airports increases the market share of a particular airport will decrease (as by assumption all airports are equal in all aspects). Hence, to break even the airport will have to increase its tax as the capital cost has not changed. The properties of the equilibrium airfares and frequencies are those of the airfares and frequencies derived in Propositions 1 and 2. The equilibrium derived in Proposition 3 is rather limited as it is necessary that airports are equal in all aspects. If e.g.  $t_d \neq t_{d'}$ , the symmetric equilibrium derived in Proposition 3 is no longer valid. In Subsection 3.4 we comment on more general equilibria.

**Extensions**

In Proposition 1 a symmetric local airline equilibrium was derived. Although this only is a special case, for our purposes (analysis of airport behavior) it is convenient to maintain the assumption that airlines are equal in all aspects. The condition that the frequency elasticity of demand has to be smaller than 1 has to be validated empirically. In Proposition 2 a global airline equilibrium was derived. At this equilibrium it is possible that different airports have charge different taxes, depending on the accessibility of the airport. In Proposition 3 a unique global

symmetric equilibrium including optimal airport taxes was derived. Asymmetric equilibria, although more likely to occur in reality, do not exist if airports (and airlines) are equal in all aspects, the airports' profit functions are non-decreasing in the output price and the frequency elasticity of demand is smaller than 1. The latter two conditions have to be validated empirically, the condition that airports (and airlines) are equal in all aspects is an assumption. If we were to relax this condition, the analytical solutions are very difficult to interpret. We could still look at the general solution as given in the proof of Proposition 1 to analyze how an airport reacts to a change in the characteristics of an alternate airport; e.g.  $\frac{\partial \text{tax}_d^*}{\partial t_d} = \frac{\partial \text{tax}_d^*}{\partial Z_0} \frac{\partial Z_0}{\partial t_d}$ .

However, as  $Z_0$  will be difficult to interpret, such an expression is most likely still very complicated and is, apart from the sign, still difficult to interpret. For a real world problem, a numerical solution for the optimal fares, frequencies and airport taxes could prove to be more fruitful. The restrictions of symmetric airlines and symmetric airports can then be relaxed. Using data for the San Francisco Bay Area, Pels et al. (1998,2) estimates for most of the necessary parameters and exogenous variables can be derived. These estimates will be used to find (asymmetric) equilibrium frequencies; data on the airfares is not available.

## CONCLUSION

Using a nested logit demand model, in this paper we have derived equilibrium airfares and frequencies. These were shown to be unique. First, a local symmetric equilibrium was derived. This is an equilibrium for airlines operating from the same airport, and provides a convenient tool for the remainder of the analysis in which optimal airfares and frequencies for airlines operating from different airports and optimal airport taxes were derived. In Proposition 2 it was shown that a symmetric equilibrium between airlines operating from different airports can exist even if the airports charge different taxes *and* have different accessibility's, although taxes and accessibility's cannot attain every value. When the airport authority is introduced as a player, symmetric airport taxes, airfares and frequencies exist and were shown to be unique given certain assumptions. When these assumptions are relaxed, the analytical solutions become very complex.

The present paper leads to the following research agenda. First, one could try to find a numerical solution rather than an analytical solution. When estimates for the parameters and data on the exogenous variables are available one could find optimal airfares, frequencies and airport taxes. Furthermore, it would be possible to analyze how these variables react to changes in the exogenous variables (e.g. construction of new roads which would change the accessibility (access times) of an airport, airport terminal or runway congestion etc.) and parameters. Second, the model could also be applied to a different choice set. In this paper, a single origin-destination market was

analyzed. The same model could also be used for a choice between direct-indirect flights, and this could be combined with e.g. an analysis of optimal airline networks. Analysis of airport access modes can also be included (by introducing a third level in the discrete choice model). Finally, the airline and airport cost functions were kept very simple, and these will have to be verified.

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## APPENDIX

This Appendix provides proofs for Propositions 1, 2 and 3.

**Proof of Proposition 1.**

Let there be  $L$  airlines operating out of airport  $d$ , of which  $L-1$  airlines (charging  $p_{j,d}$  and (each) offering a frequency  $f_{j,d}$ ) are in equilibrium. Hence we need to find an equilibrium between the  $L-1$  airlines already in equilibrium and the remaining airline  $l$ .  $P(l|d)$  can be rewritten as:

$$P(l|d) = \frac{\exp\left(\frac{\alpha_0 - \alpha_p p_{l,d} + \alpha_f \ln(f_{l,d})}{\mu_2}\right)}{\exp\left(\frac{\alpha_0 - \alpha_p p_{l,d} + \alpha_f \ln(f_{l,d})}{\mu_2}\right) + (L-1) \exp\left(\frac{\alpha_0 - \alpha_p p_{r,d} + \alpha_f \ln(f_{r,d})}{\mu_2}\right)} \quad (A1)$$

Then, if  $k_l = k_j$  and  $c_l = c_j = c$  for all  $j=1, \dots, L, j \neq l$ , the following symmetric equilibrium is found:

$$p_d^* = c + \frac{L\mu_1\mu_2}{\alpha_p(\mu_1(L-1) + \mu_2(1-P(d)))} \quad (A2)$$

$$f_d^* = \frac{1}{L} \frac{\alpha_f}{\alpha_p} \frac{P(d)N}{k} \quad (A3)$$

Hence at the local equilibrium all  $L$  airlines have the same frequency and airfare.  $p_d^*$  is decreasing in  $L$ :  $\frac{\partial p_d^*}{\partial L} = -\frac{\mu_1\mu_2(\mu_1 - \mu_2(1-P(d)))}{\alpha_p(\mu_1(L-1) + \mu_2(1-P(d)))^2}$  is negative as  $\mu_1 > \mu_2$ ; see Subsection 3.1. It is clear that  $f_d^*$  is decreasing in  $L$ .

To show that the (local) equilibrium airfare is unique, we have to show that airline  $l$ 's best response airfare function is a contraction; it should satisfy

$$\sum_{r, r \neq l}^L \left| \frac{\partial p_l^{br}}{\partial p_r} \right| < 1, \quad \text{where} \quad \left( p_l^{br} \equiv 1 + \alpha_p(p_l - c_l) \left( \frac{P(l|d)(P(d)-1)}{\mu_1} + \frac{P(l|d)-1}{\mu_2} \right) \right).$$

For simplicity we check the condition for  $L=2$ . Using the implicit function theorem and treating  $P(d)$  as exogenous yields:

$$\begin{aligned}
& \left| \frac{\alpha_p (p_i - c_i) \left( \frac{\frac{\partial P(l|d)}{\partial p_r} (P(d) - 1)}{\mu_1} + \frac{\frac{\partial P(l|d)}{\partial p_r}}{\mu_2} \right)}{\alpha_p \left( \frac{P(l|d)(P(d) - 1)}{\mu_1} + \frac{P(l|d) - 1}{\mu_2} \right) + \alpha_p (p_i - c_i) \left( \frac{\frac{\partial P(l|d)}{\partial p_i} (P(d) - 1)}{\mu_1} + \frac{\frac{\partial P(l|d)}{\partial p_i}}{\mu_2} \right)} \right| = \\
& \left| \frac{(p_i - c_i) \frac{\partial P(l|d)}{\partial p_r} \left( \frac{P(d) - 1}{\mu_1} + \frac{1}{\mu_2} \right)}{\left( \frac{P(l|d)(1 - P(d))}{\mu_1} + \frac{1 - P(l|d)}{\mu_2} \right) + (p_i - c_i) \frac{\partial P(l|d)}{\partial p_i} \left( \frac{P(d) - 1}{\mu_1} + \frac{1}{\mu_2} \right)} \right| < 1
\end{aligned} \tag{A4}$$

as the numerator is positive;  $\frac{\partial P(l|d)}{\partial p_r} = -\frac{\partial P(l|d)}{\partial p_i} > 0$  and  $\mu_1 > \mu_2$  (see Subsection 3.1). The denominator is also positive; the first term in brackets is positive as both  $P(d)$  and  $P(l|d) > 0$ . The second term is positive as  $\frac{\partial P(l|d)}{\partial p_i} < 0$ . Hence the price equilibrium is stable. Likewise, we have to show that  $\sum_{i', i' \neq i}^L \left| \frac{\partial f_i^{br}}{\partial f_{i'}} \right| < 1$ . Substituting for the optimal prices in equation 10 results in  $f_i^{br} = \frac{\alpha_f}{\alpha_p} \frac{P(d)P(l|d)}{f_i} N - k = 0$ . Using the implicit function theorem, we find:

$$\begin{aligned}
& \left| \frac{\frac{\partial P(l|d)}{\partial f_r}}{\frac{\partial P(l|d)}{\partial f_i} - \frac{P(l|d)}{f_i}} \right| = \\
& \left| \frac{-\frac{\alpha_f}{\mu_2 f_r} P(l|d)(1 - P(d))}{\frac{\alpha_f}{\mu_2 f_i} P(l|d)(1 - P(d)) - \frac{P(l|d)}{f_i}} \right| < 1
\end{aligned} \tag{A5}$$

if  $\alpha_f < \frac{\mu_2}{1 - P(l|d)}$ , as in equilibrium  $f_{i,d} = f_{i',d}$ . The condition  $\alpha_f < \frac{\mu_2}{1 - P(l|d)}$  ensures that the frequency elasticity of the (local) demand is smaller than 1;  
 $\varepsilon_{f_{i,d}}^{P(l|d)} = \frac{\partial \ln(P(l|d))}{\partial \ln(f_{i,d})} = \frac{\alpha_f}{\mu_2} (1 - P(l|d)) < 1$ . QED.

*Proof of Proposition 2.*

Suppose there are  $D$  airports, each of which accommodates  $L$  airlines. Furthermore, define  $\Delta = \beta_i(tax_{d'} - tax_d) - \beta_d(\ln(t_{d'}) - \ln(t_d))$ , where the subscript  $d'$  is used for all alternate airports; i.e. all airports other than  $d$  have equal covariates. Of the  $D$  airports in the system, let there be  $D-1$  already in equilibrium. Then  

$$P(d) = \frac{\exp(V_d)}{\exp(V_d) + (D-1)\exp(V_{d'})}$$
, where  $V_d$  is the utility derived from using airport  $d$  (as specified in equation (3)). Then, if  $L_d = L_{d'}$ , solving the system of equations (9) and (11) for both airport  $d$  and the alternate airports  $d'$ , we find a general solution:

$$\begin{aligned} p_d^* &= p_{d'}^* = c + \frac{\alpha_f \mu_1 \mu_2 L_d N}{\alpha_p (\alpha_p k \exp(Z_0) (D-1) + \mu_1 (L_d - 1) N)} \\ f_d^* &= \frac{\alpha_f N}{\alpha_p k} - \exp(Z_0) \\ f_{d'}^* &= \exp(Z_0) \end{aligned} \quad (A6)$$

where  $Z_0$  is a root of  $\exp(\Omega)^4$ ;

$$\Omega|_{L>1} = \left[ \begin{aligned} &\alpha_p^2 \exp(Z) k \left( \frac{1}{2} \Psi(\mu_1 - \alpha_f) + \Delta \right) + \alpha_f^2 \frac{1}{2} \Psi N + \\ &\left( \left( \frac{1}{2} \Psi + \frac{D}{D-1} \right) \mu_1 - \Delta \right) N \alpha_f \alpha_p \end{aligned} \right] (D-1) \exp(Z) k \mu_2^2 - \alpha_f^2 \mu_1 \mu_2^2 N^2 \quad (A7)$$

where  $\Psi = -2Z - 2\ln(k) + 2\ln(\exp(Z)\alpha_p k(1-D) + \alpha_f N) - 2\ln(\alpha_p)$ . Solving  $\Psi = 0$  results in

<sup>4</sup> For convenience we give here the expression for  $L=1$ . When  $L>1$ ,  $\Omega|_{L>1}$  would be far more complicated. However,  $Z_0$  will attain the same value. It is crucial that  $L_d = L_{d'}$ .

$$Z_0 = \ln \left( \frac{\alpha_f N}{\alpha_p Dk} \right) \quad (\text{A8})$$

Substituting equation (A8) for  $Z$  and  $\psi = 0$  in  $\Omega$  yields  $\Omega = 0$  if  $\Delta = 0$ . Substituting for  $Z_0$  in the general solution results in equations (12) and (13).

To see whether this equilibrium is stable we again check whether the best response functions are contractions. For  $D=2$ , the condition  $\sum_{d', d'' \neq d}^D \left| \frac{\partial p_{l,d}^{br}}{\partial p_{l,d'}} \right| < 1$  is:

$$\begin{aligned} & \left| \frac{(p_{l,d} - c) \frac{\partial P(l,d)}{\mu_l} \frac{\partial P(l,d)}{\partial p_{l,d'}}}{\frac{1}{\alpha_p P(l,d)} \frac{\partial P(l,d)}{\partial p_{l,d}} + (p_{l,d} - c) \frac{\partial P(l,d)}{\mu_l}} \right|_{D=2} = \\ & \left| \frac{(p_{l,d} - c) \frac{\partial P(l,d)}{\mu_l} \frac{\partial P(l,d)}{\partial p_{l,d}}}{\frac{1}{\alpha_p P(l,d)} \frac{\partial P(l,d)}{\partial p_{l,d}} + (p_{l,d} - c) \frac{\partial P(l,d)}{\mu_l}} \right| < 1 \end{aligned} \quad (\text{A9})$$

Likewise, we have

$$\begin{aligned} & \left| \frac{\frac{\partial P(l,d)}{\partial f_{l,d}}}{\frac{\partial P(l,d)}{\partial f_{l,d}} - \frac{P(l,d)}{f_{l,d}}} \right| = \\ & \left| \frac{-\frac{\alpha_f}{\mu_l f_{l,d}} P(l,d)(1 - P(d))}{\frac{\alpha_f}{\mu_l f_l} P(l,d)(1 - P(d)) - \frac{P(l,d)}{f_l}} \right| < 1 \end{aligned} \quad (\text{A10})$$

if  $\alpha_f < \frac{\mu_l}{1 - P(d)}$ . Note that  $\alpha_f < \mu_l$  as  $\alpha_f < \mu_2$  (this follows from the proof of proposition 1) and  $\mu_2 < \mu_l$  (see Subsection 3.1). QED.



**Proof of Proposition 3.**

Let the cost recovery condition be binding (i.e.  $\lambda > 0$ ). Then the airport's optimal tax is given by equation 18 and the airport's best response function is the airport's profit function. Furthermore, let  $Kap_d = Kap_{d'}$  and  $\tau \equiv t_d - t_{d'}$ . For  $L_d = L_{d'} = 2$ , the general solution to the system of equations 12, 13 and 18 is

$$\begin{aligned}
 tax_d &= \frac{\alpha_f Kap_d}{\alpha_f N - \alpha_p k Z_0 (A - 1)} \\
 tax_{d'} &= \frac{\alpha_f Kap_{d'}}{a_p k Z_0} \\
 p_d &= \frac{\alpha_p^2 c k Z_0 (A - 1) + a_f N \mu_l}{\alpha_p^2 k Z_0 (A - 1)} \\
 p_{d'} &= - \frac{\alpha_p c (a_f N - \alpha_p k Z_0) - a_f N \mu_l}{\alpha_p (a_f N - \alpha_p k Z_0)} \\
 f_d &= - \frac{-a_f N + a_p k Z_0 (A - 1)}{a_p k} \\
 f_{d'} &= Z_0
 \end{aligned} \tag{A11}$$

where  $Z_0$  is a root of  $\exp(\Omega)$ ;

$$\begin{aligned}
 \Omega &= \left[ \left( (\psi \phi_1 + (2\mu_l + \beta_a \tau) \exp(Z) \alpha_f^2 \alpha_p) k + \beta_l \alpha_f^2 r Kap_d \right) D + \right. \\
 &\quad \left. - (\psi \phi_1 + (\mu_l + \beta_a \tau) \exp(Z) \alpha_f^2 \alpha_p) k - \beta_l \alpha_f^2 r Kap_d \right] N^2 + \\
 &\quad \left[ \left( (\psi \phi_2 + (\mu_l + \beta_a \tau) \exp(Z) \alpha_f \alpha_p^2) k^2 + \beta_l \alpha_p \alpha_f^2 r Kap_d \exp(Z) \right) D^2 + \right. \\
 &\quad \left. - (\psi \phi_1 + (\mu_l + \beta_a \tau) \exp(Z) \alpha_f \alpha_p^2) k - \beta_l \alpha_p \alpha_f^2 r Kap_d \exp(Z) \right] N + \\
 &\quad \left( (\psi \phi_3 + \alpha_p^3 \beta_l \tau (\exp(Z))^3) k^3 + \alpha_p^2 \alpha_f \beta_l k^2 r Kap_d (\exp(Z))^2 \right) D^2 - \\
 &\quad \left( 2(\psi \phi_3 + \alpha_p^3 \beta_l \tau (\exp(Z))^3) k^3 + \alpha_p^2 \alpha_f \beta_l k^2 r Kap_d (\exp(Z))^2 \right) D + \\
 &\quad (\psi \phi_3 + \alpha_p^3 \beta_l \tau (\exp(Z))^3) k - \alpha_f^3 N^3 \mu_l
 \end{aligned} \tag{A12}$$

where  $\psi = -2Z - 2\ln(k) + 2\ln(\exp(Z)\alpha_p k(1-D) + \alpha_f N) - 2\ln(\alpha_p)$ ,  
 $\phi_1 = \frac{1}{2}\alpha_p^3 \alpha_f^2 \exp(Z)(\alpha_f \mu_1)$ ,  
 $\phi_2 = -\frac{1}{2}\alpha_p^2 \alpha_f (\exp(Z))^2 (\alpha_f \mu_1)$  and  
 $\phi_3 = \frac{1}{2}\alpha_p^3 (\exp(Z))^3 (\alpha_f \mu_1)$ .

Solving  $\psi=0$  yields  $Z_0 = \ln\left(\frac{1}{2} \frac{\alpha_f N}{\alpha_p D k}\right)$ . Substituting 0 for  $\psi$  in  $\Omega$  and solving  $\Omega = 0$

yields  $Z_0 = \ln\left(\frac{1}{2} \frac{\alpha_f N}{\alpha_p D k}\right)$  if  $\tau=0$ . Finally, substituting for  $Z$  in the general solution

yields equations 17, 18 and 19. To show that the symmetric equilibrium is unique we again look at the best response functions; we show this for  $D=2$ . From propositions 1 and 2 it follows that the airlines' equilibrium airfares and frequencies are unique; hence we only look at the airports' best response functions:

$$\left| \frac{\partial \Pi_d / \partial \text{tax}_d}{\partial \Pi_d / \partial \text{tax}_d} \right| = \left| \frac{(\text{tax}_d - mc_d) \frac{\beta_1}{\mu_1} P(d)(1-P(d))}{P(d) - (\text{tax}_d - mc_d) \frac{\beta_1}{\mu_1} P(d)(1-P(d))} \right| < 1 \quad (\text{A13})$$

if  $(\text{tax}_d - mc_d) \beta_1 < 2\mu_1$ . This condition implies that the airport's profit function is non-decreasing in the airport tax. The argument runs along similar lines if we look at the airport's best response to changes in airfare or frequency. Hence the equilibrium as given by equations 17, 18 and 19 is unique. QED.